Bose – Einstein distribution law ⇒

For finding – Bose – Einstein distribution let us consider a box be divided into g_i section (cell and let n_i indistreguishable particless are to be distributed among there sections

Now

The the total no of possible ways in which n_i particles can be distributed into g_i section is –

$$g_i\,(n_i + g_i - 1)! \quad \text{——} \quad (1)$$

Since, These particles are indistinguishable so there are $n_i!$ ways which corresponds to same result. hence above eq^n be divided by $n_i!$

and also the distribution arises by permutation of section among themselves do not produce different state and there are g_i such permulation so it must be divided by $g_i!$

$$\frac{g_i\,(n_i + g_i - 1)!}{g_i!\; n_i!} = \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$$

There will be similar expressions for various other quantum state.

So the total distinguishable arrangement is given by –

$$G = \frac{(n_1 + g_1 - 1)!}{n_1!\,(g_1 - 1)!} \cdot \frac{(n_2 + g_2 - 1)!}{n_2!\,(g_2 - 1)!}$$

$$\boxed{n_i = \frac{g_i}{e^{\alpha + \beta \epsilon_i} - 1}}$$

Bose Einstein distribution function

This eqn represent the most probable distribution of the particles among various energy levels for a system obeying Bose Einstein statistics.

— x —

For most probable distribution $\delta \Omega_{max} = 0$

$$\therefore \quad \Sigma \log \left(\frac{n_i}{n_i+g_i}\right) \delta n_i = 0 \quad \text{——— (3)}$$

we know

$$\delta n_i = \Sigma \delta n_i = 0 \quad \text{——— (4)}$$

$$\delta E = \delta \epsilon_i \, \delta n_i = 0 \quad \text{——— (5)}$$

Applying Lagrange method of undetermined multiples we get

$$\Sigma \left[\log \frac{n_i}{n_i+g_i} + \alpha + \beta \epsilon_i \right] \delta n_i = 0$$

δn_i are independent

$$\log \frac{n_i}{n_i+g_i} + \alpha + \beta \epsilon_i = 0$$

$$\frac{n_i}{n_i+g_i} = e^{-\alpha - \beta \epsilon_i}$$

$$\frac{n_i+g_i}{n_i} = e^{\alpha + \beta \epsilon_i}$$

$$1 + \frac{g_i}{n_i} = e^{\alpha + \beta \epsilon_i}$$

$$\frac{g_i}{n_i} = e^{\alpha + \beta \epsilon_i} - 1$$

$$\frac{n_i}{g_i} = \frac{1}{e^{\alpha + \beta \epsilon_i} - 1}$$

$$= \prod_i \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$$

A/c to postulate of equal probability, probability (Ω) of the system occurring with specific distribution is proportional to the total no of eigen states

i.e $\Omega = \text{constant} \times \prod_i \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!}$

taking log

$$\log \Omega = \text{const} + \Sigma \{ \log (n_i + g_i - 1)! - \log n_i! - \log (g_i - 1)! \}$$

1 can be neglected when compared to n_i & g_i

then

$$\log \Omega = \text{const} + \Sigma (\log (n_i + g_i)! - \log n_i! - \log (g_i)!$$

$$= \text{const} + \Sigma [(n_i + g_i) \log (n_i + g_i) - \cancel{(n_i + g_i)} - (n_i \log n_i - \cancel{n_i}) - (g_i \log g_i - \cancel{g_i})$$

$$\log \Omega = \text{const} + \Sigma [(n_i + g_i) \log (n_i + g_i) - n_i \log n_i - g_i \log g_i]$$

$$\delta \log \omega = \Sigma \left[(n_i + g_i) \times \frac{1}{(n_i + g_i)} \delta n_i + \log (n_i + g_i) \delta n_i - n_i \times \frac{1}{n_i} \delta n_i - \log n_i \, \delta n_i \right]$$

$$= - \Sigma \left(\log \frac{n_i}{n_i + g_i} \right) \delta n_i \quad \text{——} \; (2)$$